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Closed-form Solution for the Performance Characteristics of a Photovoltaic Array/DC Motor System

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Abstract *The current-voltage characteristics of the photovoltaic module are highly non-linear and depend, among other factors, on solar insolation level and on the module surface temperature. At a fixed value of insolation level to which the module is subjected, the terminal voltage decreases non-linearly as the load current increases. Most previous research in this area depends on non-linear mathematical equations relating the module voltage with its current. Therefore, numerical methods of analysis are required to determine the performance of a system in which a photovoltaic module, or array, is an element.*

It is known that most operating points in such systems lie in the range between the no-load voltage and the maximum power point voltage of the photovoltaic module.

In this article, an approach is presented to obtain approximate closed-form analytical expressions for the current-voltage characteristics of any photovoltaic array in the range from no-load condition up to the maximum power point condition. The technique presented allows the direct method to predict the operating conditions of photovoltaic arrays when feeding loads, such as DC motors, without using tedious and time-consuming iterative numerical techniques, and the approximated current-voltage characteristics obtained analytically are compared with corresponding exact characteristics, with very close agreement obtained. To validate the proposed closed-form analytical expressions of the current-voltage characteristics of the photovoltaic array when feeding DC motors, the torque speed characteristics of different types of DC motors are obtained analytically and compared with the corresponding exact characteristics. The difference between the approximate and exact characteristics is very small.

Keywords photovoltaic arrays, DC motors and analysis

1. Introduction

Recently, the use of photovoltaic (PV) power is increasingly spreading [1–3]. Among popular applications of PV arrays is their use to feed DC motor/pump systems. This is due to the simplicity of the overall system; since in this case, no DC-AC conversion, storage batteries, direct coupling, etc. are required. This arrangement is typically used in

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Nomenclature

I_a	DC motor armature current (A)
I_f	DC motor field current (A)
I_g	output current of photovoltaic (PV) array (A)
I_{MPP}	PV cell current at maximum power point (MPP) (A)
I_o	cell reverse saturation current (A)
I_{og}	array reverse saturation current (A)
I_{ph}	PV cell photocurrent (A)
I_{phg}	PV array photocurrent (A)
I_{sc}	PV module short-circuit current (A)
k	Boltzman constant (1.38×10^{-23} J/K)
K_{se}	DC separately excited motor flux coefficient
m	empirical non-idealty factor whose value is usually close to unity
M_{af}	DC series and shunt motor constant that is the mutual inductance between armature and field windings (H)
N_p	number of parallel connected strings in a PV array
N_s	number of series-connected PV modules per string
P_{MPP}	maximum power produced by a PV cell (W)
q	electron charge (1.602×10^{-19} C)
R_a	DC motor armature winding resistance (Ω)
R_f	DC motor field winding resistance (Ω)
R_s	PV cell series resistance (Ω)
R_{sg}	PV array series resistance (Ω)
T	absolute cell temperature (Kelvin)
U_g	output voltage of the PV array (V)
U_{MPP}	PV cell voltage at MPP (V)
U_{oc}	PV module open-circuit voltage (V)
Λ	PV cell factor
Λ_g	PV array factor
ω	DC motor speed (mechanical rad/s)

irrigation, where the system does not need to operate continuously and the water can be used directly or easily stored [4–6].

Many researchers have analyzed the steady-state performance of DC motors (series, separately excited, and shunt) when they are either fed from a PV array directly [7, 8] or via an intermediate converter to achieve optimal operation [9, 10]. The PV array is a power supply whose output current depends non-linearly on its terminal operating voltage. Moreover, the PV current, voltage, and power change with the solar insolation level to which the array is subjected. The current-voltage characteristic of any PV array in a system is described by a non-linear equation that causes numerical solutions to be used to obtain the required characteristics of the system.

In this article, the non-linear equation describing the current-voltage characteristic of a PV array is approximated using a second-order algebraic equation within the range between the open-circuit voltage (U_{oc}) and the MPP voltage (U_{MPP}) of the array. The coefficients of this equation are dependent on the array parameters, and it has the merit that it is easy to deal with in the analysis, leading to saving in computation time.

Also, closed-form expressions for various performance characteristics of the system are obtained when the PV array is feeding different types of DC motors, which are assumed to drive a centrifugal pump.

2. Method of Analysis

Typical characteristic curves of current and power versus voltage at a 100% insolation level for a PV module are shown in Figure 1 [11]. It is clear from this figure that there is unique point at which the power obtained from the module is maximum. This point, point (3) in Figure 1, is the MPP (P_{MPP}), and the terminal voltage decreases non-linearly as the load current increases, until it becomes zero at short circuit.

For the range between the MPP voltage corresponding to point (3), U_{MPP}, and that corresponding to the open-circuit voltage U_{oc}, point (2), the PV module current as a function of the module voltage is approximated by a second-order algebraic equation as

$$I = A + BU + CU^2 U_{MPP} \leq U \leq U_{oc}, \tag{1}$$

where A, B, and C are constant coefficients. To determine these coefficients analytically, for a certain insolation level G (Figure 1), the following approach is used.

(i) At MPP (point 1):

$$I_{MPP} = A + BU_{MPP} + CU_{MPP}^2. \tag{2}$$

(ii) At open circuit (point 2):

$$0 = A + BU_{oc} + CU_{oc}^2. \tag{3}$$

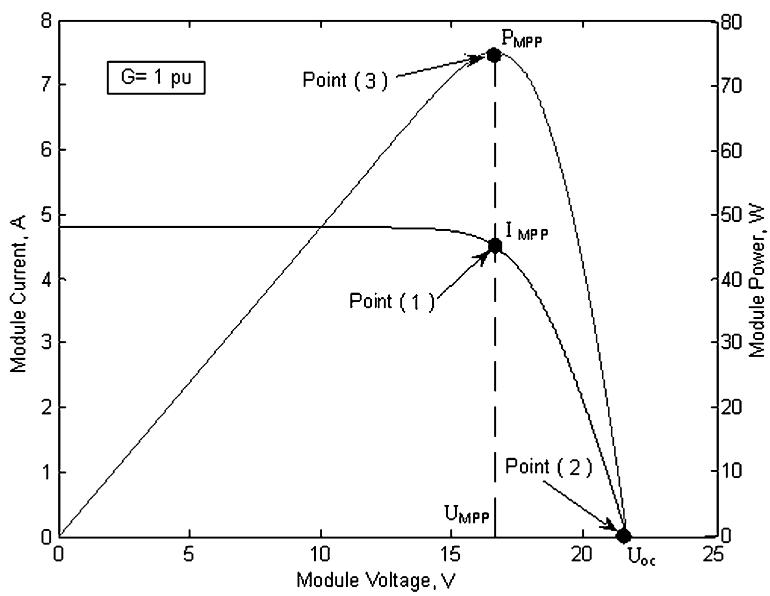

Figure 1. Current-voltage and power-voltage characteristics of a PV module.

(iii) The PV module power equation is obtained by multiplying both sides of Eq. (1) by the PV module voltage. Thus an expression of the power is obtained as

$$P = AU + BU^2 + CU^3$$

At MPP, point (3), dP/dU equals zero. Therefore,

$$0 = A + 2BU_{MPP} + 3CU_{MPP}^2. \tag{4}$$

Solving Eqs. (2)–(4) simultaneously, the coefficients A, B, and C are obtained as

$$\left.\begin{aligned} A &= (I_{MPP}/U_{MPP})[U_{oc}U_{MPP}(2U_{oc} - 3U_{MPP})/(U_{oc} - U_{MPP})^2], \\ B &= (-I_{MPP}/U_{MPP})[(U_{oc}^2 - 3U_{MPP}^2)/(U_{oc} - U_{MPP})^2], \\ C &= (I_{MPP}/U_{MPP})[(U_{oc} - 2U_{MPP})/(U_{oc} - U_{MPP})^2]. \end{aligned}\right\} . \tag{5}$$

The module voltage, in terms of the module current from Eq. (1), can be obtained as

$$U = K_1 - K_2\sqrt{K_3 + K_4 I}, \tag{6}$$

where the coefficients K_1, K_3, K_3, and K_4 are constants and function of the coefficients A, B, and C, and their expressions are given in Appendix A.

To validate the accuracy of the analytical expression (Eq. (1)), the current-voltage characteristics at several insolation levels of a module (SOLAREX module MSX-77 [11]; see Appendix B) were obtained and compared with the corresponding characteristics obtained from the exact expression relating the module current with the module voltage, which is

$$U = (1/\Lambda)\ln[1 + (GI_{ph} - I)/I_o] - IR_s. \tag{7}$$

The comparison of the two sets of results is shown in Figure 2, and the difference between the voltage values for any given module current is shown in Figure 3. From these figures, it is evident that the results obtained from the analytical approach are very close to that obtained from the exact equation (Eq. (7)). It is also noticed from Figure 3 that the maximum voltage difference between the two approaches is 0.01317 for an insolation level of 20%, with the value of this difference decreasing as the insolation level increases.

For an array composed of N_p parallel strings, with each string consisting of N_s series modules, Eq. (7) becomes

$$U_g = (1/\Lambda_g)\ln[(GI_{phg} + I_{og} - I_g)/I_{og}] - I_g R_{sg}, \tag{8}$$

where $\Lambda_g = \Lambda/N_s$, $I_{phg} = I_{ph}N_p$, $I_{og} = I_o N_p$, and $R_{sg} = R_s N_s/N_p$. A similar equation for the approximate expression of the current-voltage equation of the module in Eq. (6) can be obtained for the array as

$$U_{ga} = K_{1g} - K_{2g}\sqrt{K_{3g} + K_{4g}I_{ga}}, \tag{9}$$

where U_{ga} and I_{ga} are the approximate values of the PV array voltage and current, respectively, and the coefficients K_{1g}, K_{2g}, K_{3g}, and K_{4g} are constants, and their expressions are given in Appendix A.

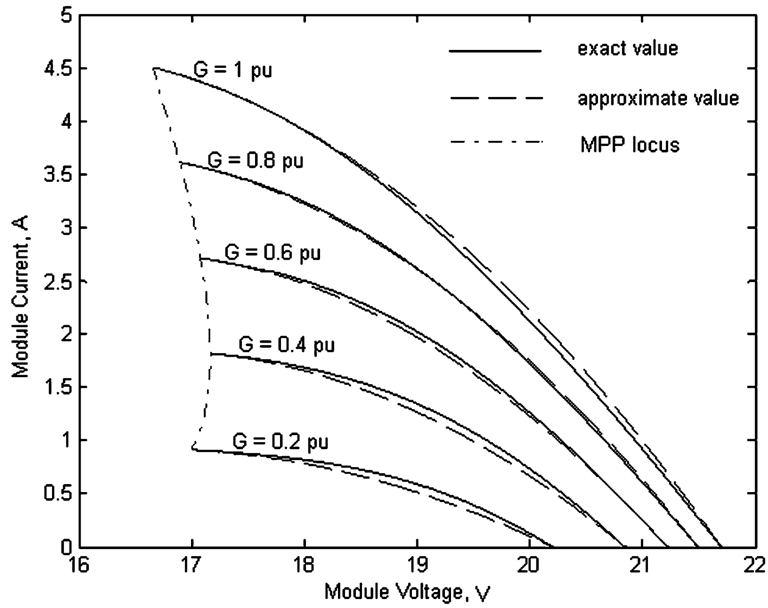

Figure 2. Comparison between exact and approximate current-voltage characteristics of a PV module at different insolation levels.

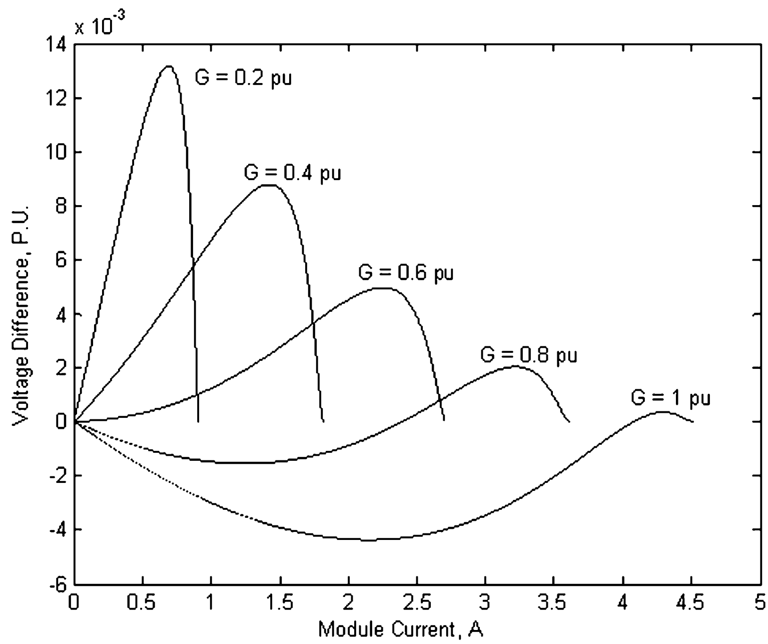

Figure 3. Voltage differences versus current of a PV module at different insolation levels.

The analytical approach derived above in Eqs. (1) or (6) will be utilized to obtain closed-form expressions for the system performance characteristics when the PV array is feeding a DC motor driving a centrifugal pump. As mentioned before, three types of DC motors will be considered, that is, series, separately excited, and shunt motor types.

2.1. PV Array—Series DC Motor System

The steady-state voltage equation of the series motor is

$$U = I_a(R_a + R_f) + M_{af} I_a \omega, \tag{10}$$

where U and I_a are the motor terminal voltage and armature current when they are equal to U_g and I_g, respectively, when the motor is fed from a PV array. The developed torque of the DC series motor is obtained from

$$T_e = M_{af} I_g^2. \tag{11}$$

When a PV array feeds the DC series motor, then the speed-torque characteristics of the motor using the exact PV array voltage in Eq. (8) can be obtained as a function of motor torque [12] as

$$\omega_{exact} = -[(R_{sg} + R_a + R_f)/M_{af}] + [1/(M_{af}\Lambda_g\sqrt{T_e/M_{af}})]$$
$$\cdot \ln[1 + [(GI_{phg} - \sqrt{T_e/M_{af}})/I_{og}]], \tag{12}$$

where ω_{exact} is the motor speed obtained when the exact PV array voltage of Eq. (8) is used. Applying the approximation of the current-voltage characteristics (Eq. (9)) to the DC series motor, a closed-form expression for the motor speed in terms of its torque can be obtained as

$$\omega_{app} = (U_{ga} - \sqrt{T_e/M_{af}}(R_a + R_f))/\sqrt{T_e M_{af}} \tag{13}$$

or

$$\omega_{app} = (K_{1g} - K_{2g}\sqrt{K_{3g} + K_{4g}\sqrt{T_e/M_{af}}} - \sqrt{T_e/M_{af}}(R_a + R_f))/\sqrt{T_e M_{af}}, \tag{14}$$

where ω_{app} is the motor speed corresponding to U_{ga}, and in this case,

$$T_e = M_{af} I_{ga}^2. \tag{15}$$

From Eq. (14), for any given value of motor torque, the motor speed can be obtained directly. On the other hand, for a given motor speed, the motor torque can be obtained by determining the motor current as a function of the motor speed employing Eqs. (9) and (10) as follows:

$$A_{sr} I_{ga}^2 + B_{sr} I_{ga} + C_{sr} = 0, \tag{16}$$

where the expressions of the coefficients A_{sr}, B_{sr}, and C_{sr} are given in Appendix A, noting that A_{sr} and B_{sr} are functions of ω_{app}.

Thus, from Eq. (16) the motor, or array, approximate current I_{ga} can be obtained as

$$\begin{aligned} I_{ga} &= [K_{1g}/(M_{af}\omega + R_a + R_f)] + [K_{2g}^2 K_{4g}/(2(M_{af}\omega + R_a + R_f)^2)] \\ &\quad - (K_{2g}/(M_{af}\omega + R_a + R_f)) \\ &\quad \cdot \sqrt{K_{3g} + (K_{1g}K_{4g}/(M_{af}\omega + R_a + R_f)) + (K_{2g}^2 K_{4g}^2/(4(M_{af}\omega + R_a + R_f)^2))}. \end{aligned} \tag{17}$$

Then, a closed-form expression of the motor torque for a given motor speed can be obtained from Eqs. (15) and (17).

2.2. PV Array—Separately Excited DC Motor System

The voltage equation of the separately excited DC motor, when fed from a PV array, is

$$U_g = I_g R_a + K_{se}\omega, \tag{18}$$

where K_{se} is the machine constant, and it is assumed to be constant since the field current is assumed constant at its rated value. The electromagnetic torque developed by the motor is given by

$$T_e = K_{se} I_g. \tag{19}$$

The exact speed of the motor from Eq. (18) and using Eq. (8) of the PV array voltage can be obtained as a function of the motor torque [12] as

$$\omega_{exact} = -[T_e(R_{sg} + R_a)/K_3^2] + [1/(\Lambda_g K_3)] \ln[1 + [(GI_{phg} - T_e/K_3)/I_{og}]]. \tag{20}$$

A closed-form expression for the motor speed in terms of its torque can be obtained using the approximate PV array voltage U_{ga} (Eq. (9)) as

$$\omega_{app} = (K_{se}(K_{1g} - K_{2g}\sqrt{K_{3g} + K_{4g}T_e/K_{se}}) - R_a T_e)/K_{se}^2. \tag{21}$$

On the other hand, for a given motor speed, the motor torque can be obtained by determining the motor current as a function of the motor speed employing Eqs. (9) and (18) as follows:

$$A_{se} I_{ga}^2 + B_{se} I_{ga} + C_{se} = 0, \tag{22}$$

where the expressions of the coefficients A_{se}, B_{se}, and C_{se} are given in Appendix A, noting that B_{se} and C_{se} are functions of ω_{app}.

Equation (22) is a quadratic equation in the aramture current of the motor, from which we get

$$\begin{aligned} I_{ga} &= [-(K_{se}\omega - K_{1g}) + K_{2g}^2 K_{4g}]/R_a \\ &\quad + 1/(2R_a^2)\sqrt{4R_a^2 K_{2g}^2 K_{3g} + K_{2g}^4 K_{4g}^2 - 4R_a K_{2g}^2 K_{4g}(K_{se}\omega - K_{1g})}. \end{aligned} \tag{23}$$

Thus, a closed-form expression for the motor torque in terms of peed is obtained, using Eq. (19) with I_g replaced by I_{ga}, as

$$T = K_{se}[-(K_{se}\omega - K_{1g}) + K_{2g}^2 K_{4g}]/R_a$$
$$+ K_{se}/(2R_a^2)\sqrt{4R_a^2 K_{2g}^2 K_{3g} + K_{2g}^4 K_{4g}^2 - 4R_a K_{2g}^2 K_{4g}(K_{se}\omega - K_{1g})}. \quad (24)$$

2.3. *PV Array—Shunt DC Motor System*

The steady-state armature current and voltage equation of the shunt motor is

$$U = I_a R_a + M_{af} I_f \omega, \quad (25)$$

and the corresponding equation for the field is

$$U = I_f R_f. \quad (26)$$

The motor line current I is given by

$$I = I_a + I_f. \quad (27)$$

The motor torque is obtained from

$$T_e = M_{af} I_a I_f. \quad (28)$$

U and I are the shunt motor terminal voltage and line current, which are equal to U_g and I_g, respectively, when the motor is fed from the PV array. Thus, for a certain value of the motor line current, using Eqs. (25), (26), and (28), the speed-torque relationship of the motor using the exact PV array voltage (Eq. (8)) can be obtained as [12]

$$\sqrt{T_e R_a R_f^2 / M_{af}(R_f - M_{af}\omega_{ex})}$$
$$= -[(R_a + R_f - M_{af}\omega_{ex})\sqrt{R_{sg}^2 T_e / [R_a M_{af}(R_f - M_{af}\omega_{ex})]}$$
$$+ (1/\Lambda_g)\ln[1 + [(G I_{phg} - I)/I_{og}]]. \quad (29)$$

However, for any given value of the motor line current, the motor voltage is obtained from Eq. (8). Therefore, the field current and armature current is obtained from Eqs. (26) and (27), respectively. Thus, the motor torque is obtained from Eq. (28), and consequently, the motor speed can be determined.

In order to obtain a closed-form speed-torque characteristic using the approximate PV array current-voltage equation (Eq. (1)), a third-order voltage equation, as a function of the motor torque, can be derived from Eqs. (1) and (26)–(28) as

$$U^3 + [(B/C) - 1/(R_f C)]U^2 + (A/C)U - T_e R_f / M_{af} C = 0. \quad (30)$$

This equation is solved to determine the appropraite voltage root value as

$$U = A_{sh} + J B_{sh}, \quad (31)$$

where the expressions of the coefficients A_{sh} and B_{sh} are given in Appendix A. Thus, the motor field current is obtained from Eq. (26), the motor armature current is obtained from Eq. (28), and the motor speed is obtained from Eq. (25).

2.4. Equivalent Resistance of a PV Array–DC Motor System

The above systems can be represented by a PV array feeding a resistive load, since the DC motor is drawing a DC current as the resistive load does when fed from the PV array. This resistance is named the equivalent resistance R_{eq} of the system. Then using the exact and approximate expressions of the array voltage in terms of its current, we get

$$R_{eq\text{-}exact} = U_g / I_g \tag{32}$$

for the exact expression, and

$$R_{eq\text{-}app} = U_{gap} / I_{ga} \tag{33}$$

for the approximate expression.

3. Results

The method of analysis presented in this article is applied to a PV array consisting from a SOLAREX PV module and series, separately excited, and shunt DC motors whose parameters are given in Appendix B.

To satisfy the load (DC motor) requirements, the PV array is sized in such a manner that the number of series-connected modules in one string, N_s, is equal to eight, and the number of parallel strings N_p is four.

The speed-torque characteristics for the motors are obtained for both cases—one using the exact voltage equation, and the other using the approximate voltage equation; the two sets of results are found to be in close agreement. Figure 4 shows the results for the series motor using Eqs. (12) and (14), Figure 5 shows the results for the separately

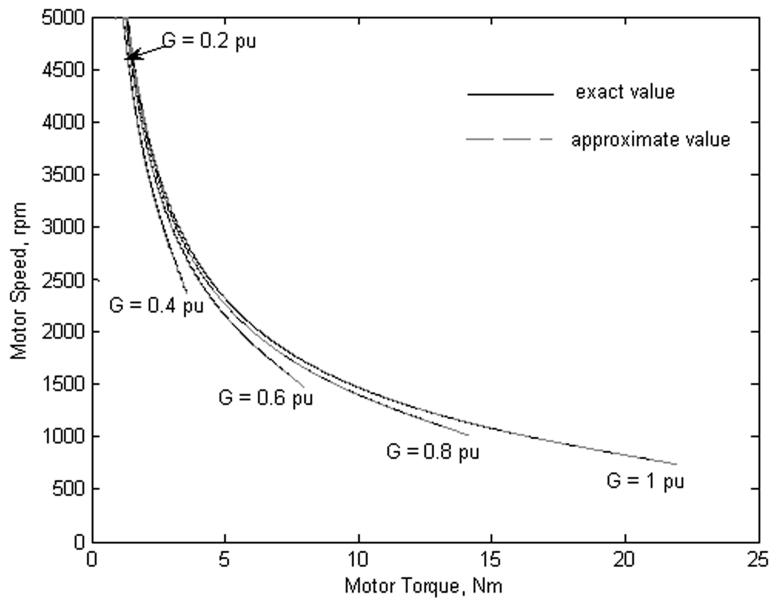

Figure 4. Speed-torque characteristics of the series DC motor.

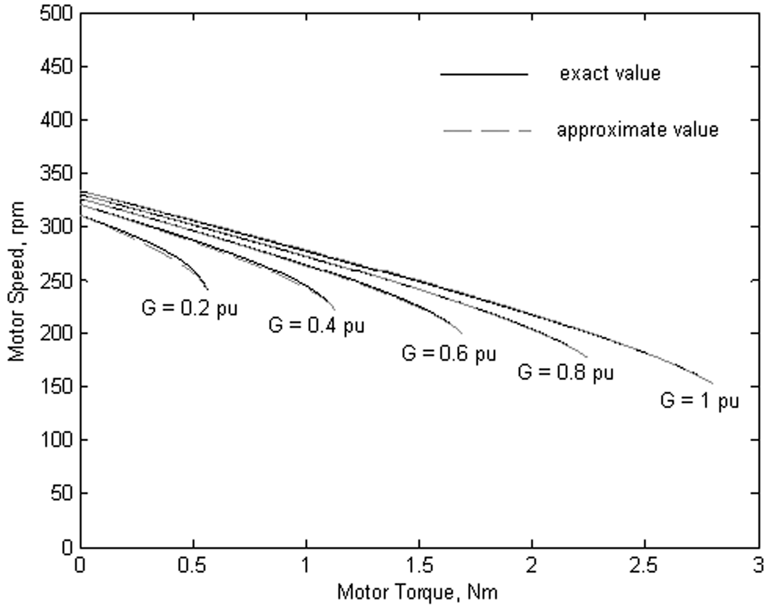

Figure 5. Speed-torque characteristics of the separately excited DC motor.

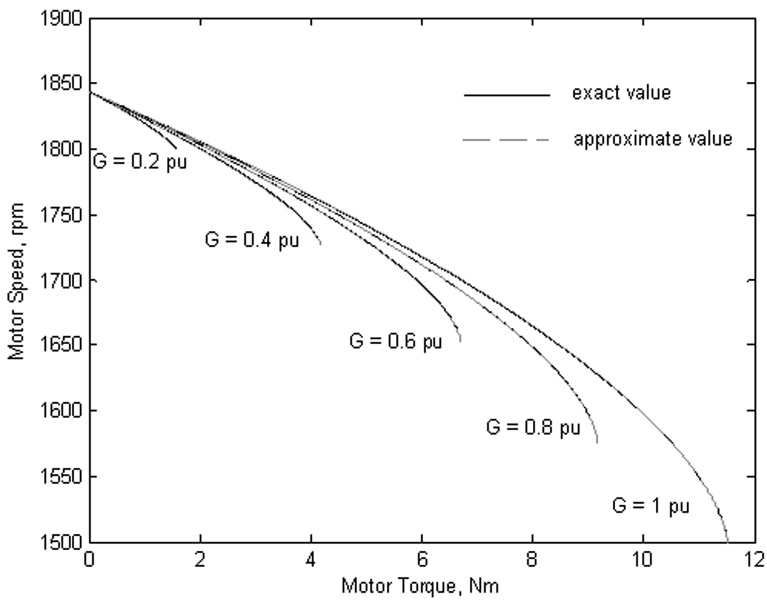

Figure 6. Speed-torque characteristics of the shunt DC motor.

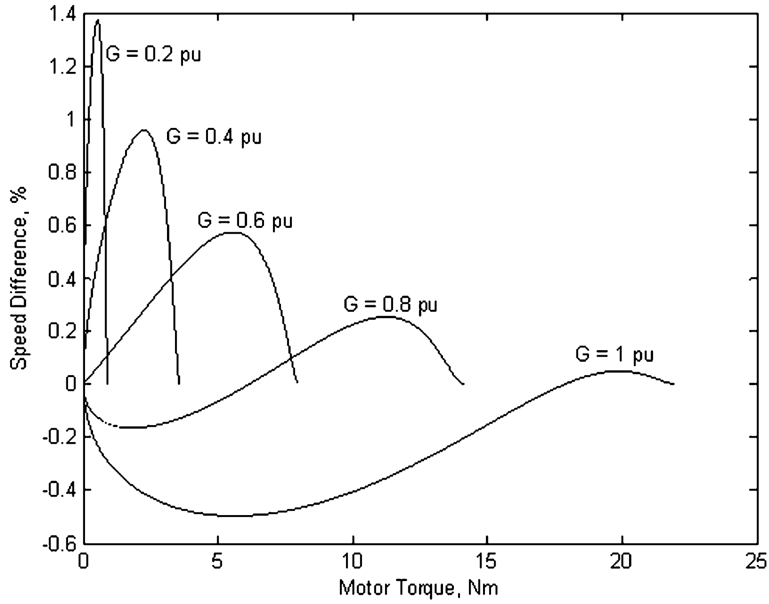

Figure 7. Speed difference-torque characteristics of the series DC motor.

excited motor using Eqs. (20) and (21), and Figure 6 shows the results for the shunt motor using Eqs. (29) and (30). For all cases, five values of insolation levels were used, which are $G = 1.0$ p.u., 0.8 p.u., 0.6 p.u., 0.4 p.u., and 0.2 p.u.

To illustrate the accuracy of the approach presented in this article, the difference of the motor speed when using the exact and approximate PV array voltage is drawn versus the motor torque for the series, separately excited, and shunt DC motors, as shown in Figures 7, 8, and 9, respectively. It is evident from these figures that the accuracy of this approach is high, especially for the case of the shunt motor.

4. Verification of Results

Previously published exact results values [7] for another PV array has been used to verify the approach of the previously discussed analysis. The data of the PV cells used are $R_s = 0.05\ \Omega$, $I_{ph} = 0.756$ A, $I_o = 0.45 \times 10^{-3}$ A, $\Lambda = 13.68$ 1/V, and the insolation level used is 1000 W/m^2. The parameters of the motor used are given in Appendix B. The number of series modules in each string of the PV array used in the above-mentioned reference is 9 and 36 cells in each module. The number of parallel strings is 18.

The approach proposed in reference [7] was used, and Figure 10 shows the comparison between the speed-torque characteristics for the series motor for several insolation levels. Figure 11 shows the comparison between the speed-torque characteristics for the separately excited motor for several insolation levels. Figure 12 shows the comparison between the speed-torque characteristics for the shunt motor for several insolation levels. In all of these figures, only the portion of the characteristics from no-load to the condition corresponding to MPP operation is displayed. Also from these figures, it is evident that the results obtained from the proposed approach and from [7] are identical.

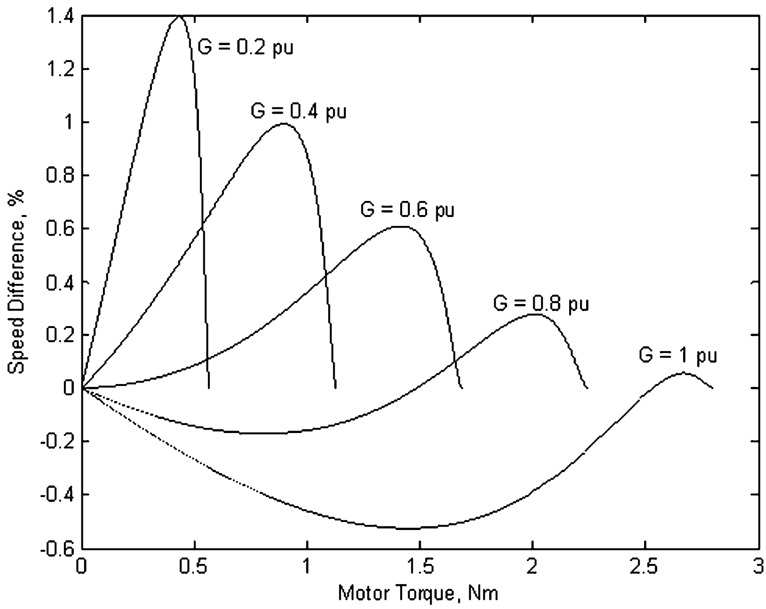

Figure 8. Speed difference-torque characteristics of the separately excited DC motor.

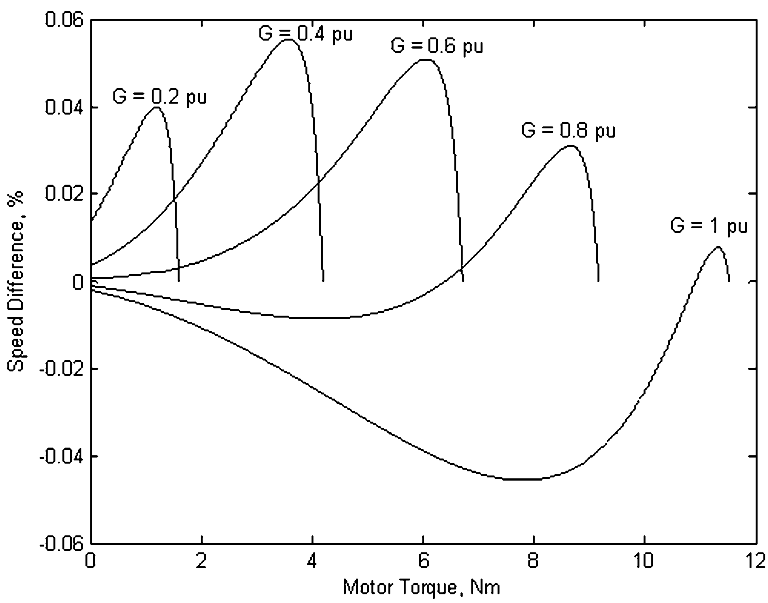

Figure 9. Speed difference-torque characteristics of the shunt DC motor.

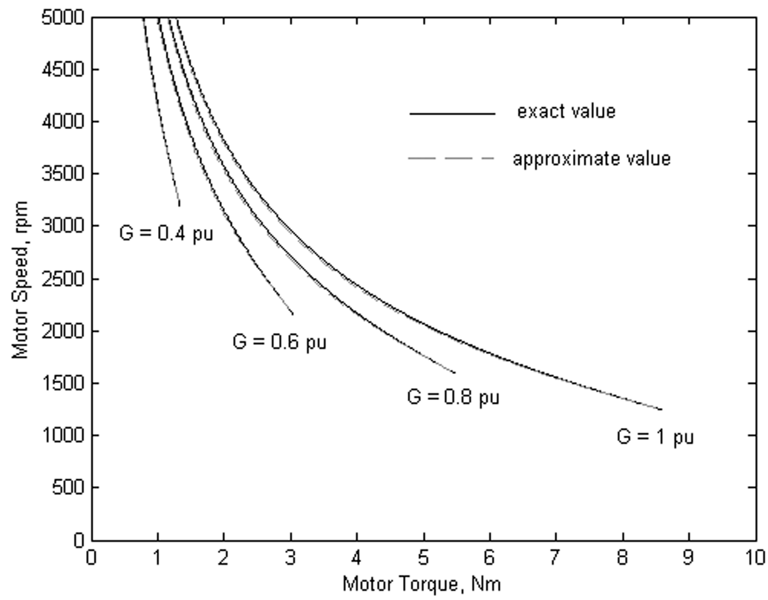

Figure 10. Speed-torque characteristics of the series DC motor.

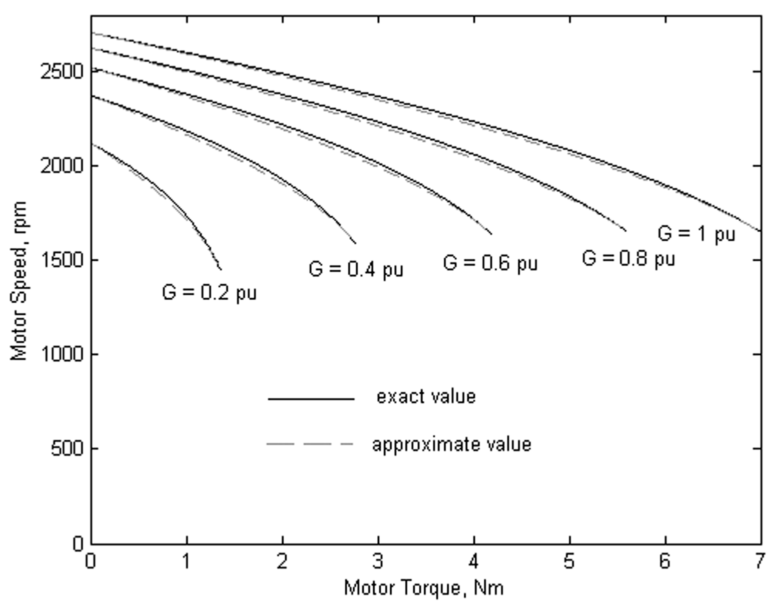

Figure 11. Speed-torque characteristics of the separately excited DC motor.

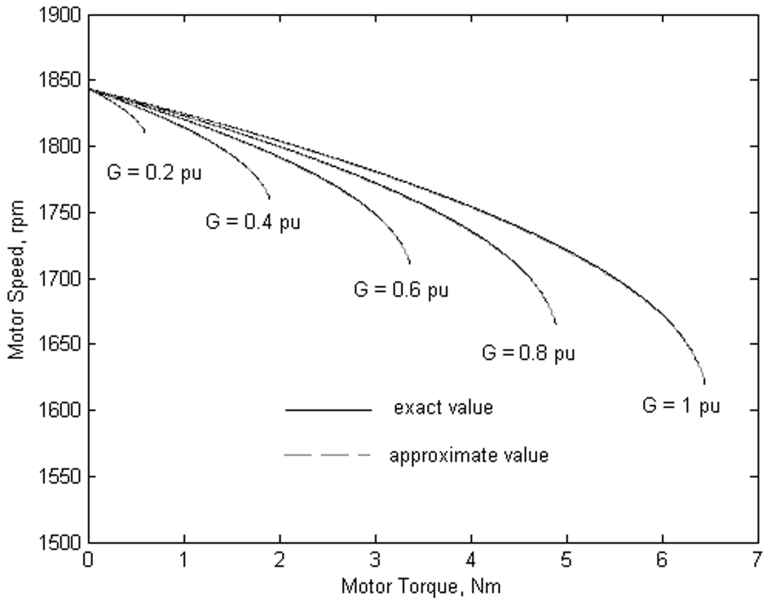

Figure 12. Speed-torque characteristics of the shunt DC motor.

5. Conclusion

A new technique to obtain approximate closed-form expressions for the current-voltage characteristics of any PV array in the range between the no-load condition and the MPP condition has been presented. The approximated current-voltage characteristics are compared with the corresponding exact characteristics using a MATLAB software programming package, and very close agreement was obtained. Closed-form expressions for the torque speed characteristics of different types of DC motors fed from a PV array are obtained. These characteristics are compared with their corresponding exact characteristics, and the difference is very small.

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Appendix A

Coefficients K_1, K_2, K_3, and K_4 of a PV module (Eq. (6)):

$$K_1 = (U_{oc}^2 - 3U_{MPP}^2)/(2U_{oc} - 4U_{MPP}),$$

$$K_2 = (U_{oc} - U_{MPP})/(2U_{oc} - 4U_{MPP}),$$

$$K_3 = (U_{oc} - 3U_{MPP})^2,$$

$$K_4 = 2U_{MPP}(2U_{oc} - 4U_{MPP})/I_{MPP}.$$

Coefficients K_{1g}, K_{2g}, K_{3g}, and K_{4g} of a PV array (Eq. (9)):

$$K_{1g} = (U_{ocg}^2 - 3U_{MPPg}^2)/(2U_{ocg} - 4U_{MPPg}),$$

$$K_{2g} = (U_{ocg} - U_{MPPg})/(2U_{ocg} - 4U_{MPPg}),$$

$$K_{3g} = (U_{ocg} - 3U_{MPPg})^2,$$

$$K_{4g} = 2U_{MPPg}(2U_{ocg} - 4U_{MPPg})/I_{MPPg},$$

where $U_{MPPg} = N_s U_{MPP}$ and $U_{ocg} = N_s U_{oc}$. Coefficients A_{sr}, B_{sr}, and C_{sr} of a PV array–DC series motor system (Eq. (16)):

$$A_{sr} = (M_{af}\omega + R_a + R_f)^2/K_{2g}^2,$$

$$B_{sr} = [2K_{1g}(M_{af}\omega + R_a + R_f)/K_{2g}^2] + K_{4g},$$

$$C_{sr} = (K_{1g}/K_{2g})^2 - K_{3g}.$$

Coefficients A_{se}, B_{se}, and C_{se} of a PV array–DC separately excited motor system (Eq. (22)):

$$A_{se} = R_a^2,$$

$$B_{se} = 2R_a(K_{se}\omega - K_{1g}) - K_{2g}^2 K_{4g},$$

$$C_{se} = (K_{se}\omega - K_{1g})^2 - K_{2g}^2 K_{3g}.$$

Coefficients A_{sh} and B_{sh} of a PV array–DC shunt motor system (Eq. (31)):

$$A_{sh} = -\sqrt[3]{K_{sh}}/(12CR_f M_{af}) + [(3ACR_f^2 - B^2R_f^2 + 2BR_f - 1)M_{af}]/(3\sqrt[3]{K_{sh}C}R_f)$$
$$- (BR_f - 1)/(3CR_f),$$

$$B_{sh} = -\sqrt{3}\{\sqrt[3]{K_{sh}}/(12R_f CM_{af})$$
$$+ [(3ACR_f^2 - B^2R_f^2 + 2BR_f - 1)M_{af}]/(3\sqrt[3]{K_{sh}C}R_f)\},$$

where

$$K_{sh} = M_{af}^2 \Big\{36ABCR_f^3 M_{af} - 36ACR_f^2 M_{af} + 108T_m C^2 R_f^4 - 8B^3R_f^3 M_{af}$$
$$+ 24B^2R_f^2 M_{af} - 24BR_f M_{af} + 8M_{af} + 12\sqrt{3}CR_f^2$$
$$\cdot [4A^3CR_f^2 M_{af}^2 - A^2B^2R_f^2 M_{af}^2 - A^2M_{af}^2 + 2A^2BR_f M_{af}^2$$
$$- 18ACR_f^2 M_{af} T_m + 18ABCR_f^3 M_{af} T_m + 12B^2R_f^2 M_{af} T_m$$
$$-4B^3R_f^3 M_{af} T_m + 27C2R_f^4 T_m^2 - 12BR_f M_{af} T_m + 4M_{af}T_m]^{0.5}\Big\}.$$

Appendix B

Parameters of SOLAREX PV Module (MSX-77/12V) used in the array:

maximum power rating $P_{\max} = 77$ W
minimum power rating $P_{\max} = 72$ W
rated current $I_{MPP} = 4.56$ A
rated voltage $U_{MPP} = 16.9$ V
short-circuit current $I_{SC} = 5.0$ A
open-circuit voltage $U_{OC} = 21.0$ V
$\Lambda_m = 1.0815$
$R_{sm} = 0.5465\ \Omega$
$I_o = 6.8398 \times 10^{-10}$ A
$I_{ph} = 5.0$ A

Parameters of the DC series motor:

$U = 120$ V
$I_a = 9.2$ A
$R_a = 1.5\ \Omega$
$R_f = 0.7\ \Omega$
$M_{af} = 0.0675$ H

Parameters of the DC separately excited motor:

$U = 120$ V
$I_a = 9.2$ A
$R_a = 1.5\ \Omega$
$K_{se} = 0.621$ V/rad/sec

Parameters of the DC shunt motor:

$U = 120$ V
$I_a = 9.2$ A
$R_a = 1.5\ \Omega$
$R_f = 100\ \Omega$
$M_{af} = 0.518$ H